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23rd YUGOSLAV CONGRESS OF THEORETICAL AND APPLIED MECHANICS (JUMEH 1999 ==> 2001)

The 23rd Yugoslav Congress of Theoretical and Applied Mechanics was originally supposed to be held in June 1999 in Bečići, Montenegro, Yugoslavia, and organized by the Yugoslav Society of Mechanics (YSM).

The members of the Scientific Committee were (in alphabetical order): Nikola Hajdin, Head of the Committee (Yu), Dimitry Beskos (Greece), Luis Bevilacqua (Brazil), Zoran Boričić (Yu), Ranislav Bulatović (Yu), Henry Busby (USA), Vladan Djordjević (Yu), Djordje Djukić (Yu), Franz Durst (Germany), E.E. Gdoutos (Greece), Katica (Stevanović) Hedrih (Yu), Richard Hetnarski (USA), Jovo Jarić (Yu), John Katsikadelis (Greece), Miloš Kojić (Yu), Anthony Kounadis (Greece), Valerii Kozlov (Russia), Dusan Krajinovic (USA), Horst Lippmann (Germany), Vlado Lubarda (USA), Milan Mićunović (Yu), Ingo Müller (Germany), Ekkehard Ramm (Germany), Valentin Rumyantsev (Russia), Dobroslav Ružić (Yu), Jean Salençon (France), Viktor Saljnikov (Yu), Werner Schiehlen (Germany), Miodrag Sekulović (Yu), Hanz Troger (Austria), Natalija Naerlović-Veljković (Yu), Božidar Vujanović (Yu), Veljko Vujičić (Yu), Luka Vujošević (Yu), Petar Vukoslavčević (Yu), Jurgen Zierep (Germany), Djordje Zloković (Yu) and Henryk Zorski (Poland).

More than 300 papers were submitted, between them 75 from abroad.

In March 1999, the country was attacked and bombed and the unanimous decision of the Bureau of the YSM was to postpone but not to cancel the Congress.

Later, in June 2001, the Bureau decided to organize the Postponed Congress (JUMEH 1999 ==> 2001) in Belgrade from October 12 to 14, 2001.

That time the General lectures were offered to invited foreign scientists and the short contributions were presented in three sections.

The Proceedings contain the contributions prepared and sent in electronic form by the authors after the Congress. They are presented in the alphabetical order.

Aleksandar Simonović, Djordje Čantrak and Nina Andjelić, helped in the preparation of the Proceedings.

D. Ružić

Sponsored by Serbian Ministry of Science, Technology and Development.

SAME PROBLEMS OF IONIZED GAS FLOW IN THE BOUNDARY LAYER FOR DIFFERENT FORMS OF THE ELECTROCONDUCTIVITY CHANGE LAW

Branko Obrović, Slobodan Savić

1. Introductory studies, starting equations

In MHD boundary layer theory, different problems of a body rounded with conductive, incompressible fluid are studied in details (by application of the parametric method). In the studies considering this field [2, 3] flow problems in the cases when the external magnetic field is perpendicular to the contour of the body are solved. Here, the fluid electroconductivity is either constant or different forms of the electroconductivity change law [3] are used. In these mentioned (and other) studies, the importance of electroconductivity variation for practical usage as well as for theory and methodology is stressed.

In that sense, this paper studies ionized gas flow in the boundary layer at the body of any shape for a few different possible forms of the law of its electroconductivity. As a matter of fact, since we do not know the exact form of the electroconductivity change law of the gas σ , in this paper we assume by analogy with the electroconductive liquid that the electroconductivity change law of the ionized gas σ is determined by the expressions:

$$(a) \quad \sigma = \sigma(x), \quad (b) \quad \sigma = \sigma_0(1 - u/u_e), \quad (c) \quad \sigma = \sigma_0 \frac{v_0}{u_e^2} \frac{\partial u}{\partial y}, \quad (\sigma_0, v_0 = const.). \quad (1)$$

As it is seen in the given presumed forms of the law, the ionized gas electroconductivity is: a function only of the longitudinal coordinate x (the expression 1 a), a function of the relation of the velocities (the case 1 b), or a function of the longitudinal velocity gradient (1 c). Based on the form of the law (1 b) and the velocity (1 c), it is concluded that in these cases the ionized gas electroconductivity disappears at the outer boundary of the boundary layer (for which $u = u_e(x)$ and $\partial u / \partial y = 0$), that is, $\sigma = \sigma_e = 0$ at this boundary.

If by the usual procedure we exclude the pressure from the starting boundary layer equations [4], the equations of the steady plane laminar boundary layer, for small values of magnetic Reynolds number and under the conditions of so-called balanced ionization, and using the usual symbols, have the following form:

$$\begin{aligned}
& \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \\
& \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \underline{\sigma B_m^2 u}, \\
& \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u \rho_e u_e \frac{du_e}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \frac{\partial h}{\partial y} \right) + \underline{\sigma B_m^2 u^2}; \quad (2) \\
& y = 0: \quad u = v = 0, \quad h = h_w; \quad y \rightarrow \infty: \quad u \rightarrow u_e(x), \quad h \rightarrow h_e(x).
\end{aligned}$$

In these equations, the member $\underline{\sigma B_m^2 u}$ represents Lorenz force, and the member $\underline{\sigma B_m^2 u^2}$ represents Jule's heat. Since the ionized gas electroconductivity σ appears only in these members, they are different for different forms of the electroconductivity change law (1). Therefore these members are underlined in the equations (2). The boundary layer equations (2) correspond to the electroconductivity change laws (1 b) and (1 c). In the case when $\sigma = \sigma(x)$ the underlined members have these forms in the dynamic and energy equation:

$$(a) \quad \underline{+ \sigma B_m^2 (u_e - u)}, \quad \underline{+ \sigma B_m^2 (u^2 - uu_e)}. \quad (2')$$

2. Generalized equations of the ionized gas boundary layer and their parametric solutions

While solving the differential equation systems (2) and (2'), polyparametric method [6] has been used.

As with similar flow problems [5] purposeful similarity transformations and parameter sets like Loitsianskii's are introduced in this paper. During the studies it has been determined that the corresponding sets of dynamic and magnetic parameters have different forms and that they satisfy different recurrent simple differential equations in agreement with the laws (1).

In the so-called three-parametric, twice localized approximation ($\partial/\partial\kappa = 0, \partial/\partial g_1 = 0$) the starting equation system (2), after extremely complicated transformations, comes down to:

$$\begin{aligned}
& \frac{\partial}{\partial \eta} \left[Q \frac{\partial^2 \phi}{\partial \eta^2} \right] + \frac{aB^2 + (2-b)f_1}{2B^2} \phi \frac{\partial^2 \phi}{\partial \eta^2} + \frac{f_1}{B^2} \left[\frac{\rho_e}{\rho} - \left(\frac{\partial \phi}{\partial \eta} \right)^2 \right] - \frac{g_1}{B} \frac{\partial \phi}{\partial \eta} \frac{\partial^2 \phi}{\partial \eta^2} = \\
& = \frac{F_m f_1}{B^2} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial^2 \phi}{\partial \eta \partial f_1} - \frac{\partial \phi}{\partial f_1} \frac{\partial^2 \phi}{\partial \eta^2} \right),
\end{aligned}$$

$$\begin{aligned}
 & \frac{\partial}{\partial \eta} \left(\frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} \right) + \frac{aB^2 + (2-b)f_1}{2B^2} \phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f_1}{B^2} \frac{\rho_e}{\rho} \frac{\partial \phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^2 \phi}{\partial \eta^2} \right)^2 + \\
 & + \underline{\frac{2\kappa g_1}{B} \left(\frac{\partial \phi}{\partial \eta} \right)^2 \frac{\partial^2 \phi}{\partial \eta^2}} = \underline{\frac{F_m f_1}{B^2} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_1} - \frac{\partial \phi}{\partial f_1} \frac{\partial \bar{h}}{\partial \eta} \right)}; \\
 & \eta = 0: \phi = \frac{\partial \phi}{\partial \eta} = 0, \bar{h} = \bar{h}_w = \text{const.}; \quad \eta \rightarrow \infty: \frac{\partial \phi}{\partial \eta} \rightarrow 1, \bar{h} \rightarrow \bar{h}_e = 1 - \kappa.
 \end{aligned} \tag{3}$$

The equation system (3) is valid in the case when the gas electroconductivity changes according to the law (1 c). For the change laws (1 a) and (1 b), the underlined members change and have slightly different form. In our studies, while solving the equation system (3), the exact expression for the function F_m has been used; of course according to the corresponding form of the electroconductivity change law. And in this paper, by analogy with dissociated gas [7], the following pretty rough dependences are used for the non-dimensional function Q and for the relation of the densities ρ_e/ρ :

$$Q = Q(\bar{h}) \approx \left(\frac{\bar{h}_w}{\bar{h}} \right)^{1/3}, \quad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1 - \kappa}; \tag{4}$$

while the usual values are accepted for Prandtl's number and the constants a and b :
 $Pr = 0,712$ (air); $a = 0,4408$; $b = 5,7140$.

Using the finite differences method, the parametric equation system (3) has been numerically solved for a few different values of the parameters κ and g_1 , given in advance, as well as for the enthalpy \bar{h}_w . Out of the two obtained results in a form of tables, only some, in a form of the corresponding diagrams (Fig. 1 and Fig. 2) are given here, and those are for different forms of the ionized gas electroconductivity law.

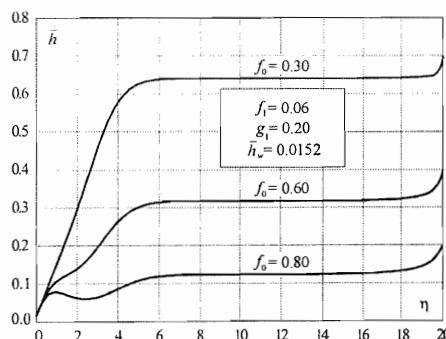


Fig. 1. Distribution of the non-dimensional Enthalpy for different values of the parameter

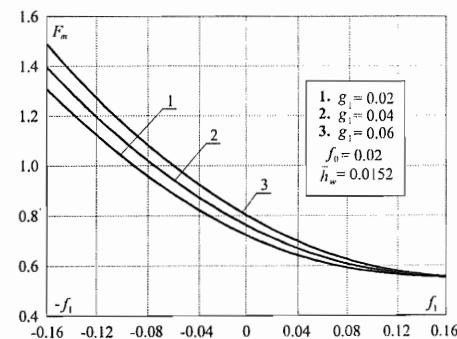


Fig. 2. The characteristic of the boundary layer

3. Conclusions

Based on the given and other diagrams, the following has been concluded:

- The non-dimensional velocity u/u_e converges very fast to zero for all the values of the compressibility parameter and different values of the form parameter and the magnetic parameter.
- Here the compressibility parameter κ has a negligible influence on distribution of the non-dimensional velocity in the boundary layer.
- This parameter has a great influence on distribution of the enthalpy in the boundary layer.

It is pointed out that the given conclusions are in total agreement with the conclusions, which are valid for similar compressible fluid flow problems (dissociated gas). These conclusions are valid for all three forms of ionized gas electroconductivity change law. We expect that the behaviour of the physical values and functions would be the same with the other laws, i.e. with the exact electroconductivity change law.

References

- [1] T. Watanabe and I. Pop, Thermal boundary layer in magnetohydrodynamic flow over a flat plate in the presence of a transverse magnetic field, *Acta Mech.*, 105, (1994), 233 - 238.
- [2] Z. Boricic, D. Nikodijevic and B. Obrovic, Unsteady flow of liquid whose electroconductivity is a function of longitudinal velocity gradient in MHD boundary layer at the body (in Serbian), Proceedings of the YUCTAM, 20th. Yugoslav Congress of Theoretical and Applied Mechanics, Kragujevac, Yugoslavia, (1993), 136-139.
- [3] V. Salnikov, Z. Boricic and D. Nikodijevic, Parametric method in unsteady MHD boundary layer theory of fluid with variable electroconductivity, *Facta Universitatis*, Vol. 2, № 7/2, (1997), 331-340.
- [4] L.G. Loitsianskii, Laminar boundary layer (in Russian), Fizmatgiz, Moscow, 1962.
- [5] B. Obrovic, Boundary layer of disassociated gas (in Serbian), Monograph, Kragujevac, University of Kragujevac, Faculty of Mechanical Engineering, 1994.
- [6] V. Salnikov and U. Dallmann, Verallgemeinerte Ähnlichkeitslösungen für dreidimensionale, laminare, stationäre, kompressible Grenzschichtströmungen an schiebenden profilierten Zylindern, Institut für Theoretische Strömungsmechanik, DLR-FB 89-34, Göttingen, 1989.
- [7] N. V. Krivtsova, Parameter method of solving of the laminar boundary layer equations with axial pressure gradient in the conditions of balance dissociation (in Russian), *Engineering-Physical Journal*, X (2), (1966), 143-153.

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